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Electromechanical wave in power systems: theory and applications

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Abstract The continuum model is a key paradigm describing the behavior of electromechanical transients in power systems. In the past two decades, much research work has been done on applying the continuum model to analyze the electromechanical wave in power systems. In this work, the uniform and non-uniform continuum models are first briefly described, and some explanations borrowing concepts and tools from other fields are given. Then, the existing approaches of investigating the resulting wave equations are summarized. An application named the zero reflection controller based on the idea of the wave equations is next presented.

Keywords Power system, Electromechanical wave propagation, Continuum model, Zero reflection controller

1 Introduction

The electromechanical wave theory introduces a set of models and analytical tools, which provide explanatory and predicting power on how the electromechanical disturbances propagate in a power system. Unlike the traditional method of studying the electromechanical transient phenomena by differential algebraic equations, the electromechanical wave theory introduces partial differential equations known as wave equations to model the transient process in power networks. Obviously, the solution of the partial differential equations exhibits travelling wave behaviour. By doing this, the concepts and tools in the research field of wave equations can be applied to the analysis of electromechanical transients in power systems. This theory casts a light on the global understanding of power system transient behaviour, and can also explain the wave-like behaviour with velocity greatly less than that of light observed in power systems by phasor measurement units (PMUs) [1].

The propagation of electromechanical disturbances, considered as travelling waves, first appeared in 1974. In [2], the second-order linear hyperbolic wave equation is derived with constant parameters from a swing equation in the context of the idealized continuum limit. Here, Semlyen took the power network as a homogeneous and isotropic system with lossless transmission lines and generators reduced to voltage sources behind a constant reactance. Cresap and Hauer [3] went further in 1981, studying what would emerge from the Western Power System in USA with the continuum model. By comparing observed data of the actual system with the model frequencies obtained from the simple wave equations, they conjectured that the continuum model can provide information about the low frequency modes of the system. In Dersin and Levis [4] the homogeneity and the isotropy constraint are relaxed by dividing the network into cells and averaging the system parameters within each cell. His resulting description is a second-order linear elliptic equation. In Thorp et al. [5] reconstructed the similar model in a simple case consisting of generators and transmission lines with the same per-unit length line

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impedances. In their following work [6], they employed many assumptions made by Semlyen and Dersin into account. These additional factors appear in the continuum model as nonlinear terms, which is indispensable in discussing the disturbance propagation velocities and their stability. In [7], the derivation of the continuum model is presented, making an analogy between power networks and multi-mass disks. In [8], the approximately analytical solutions of the non-uniform continuum model are obtained by borrowing the method of wave propagation in non-uniform media, the so-called Wentzel-Kramers-Brillouin-Jeffery (WKBJ).

The organization of this paper is as follows: Sect. 2 summarizes the basic concepts and derivation of various versions of continuum models. Sect. 3 is devoted to the analysis tools of continuum equations. Section 4 presents the application of the zero reflection controller based on continuum equations. Some open questions are outlined and future research areas prospected in Sect. 5.

2 Uniform and non-uniform continuum models

2.1 Thorp's version

2.1.1 The uniform version

In [5], J. S. Thorp et al. discussed how to derive the continuum model. They preceded this using the simplest network depicted in Fig. 1. This network consists of generators interconnected with transmission lines having a series resistance-inductance impedance.

Applying the Kirchoff's current law at node (x, y), the following equation can be obtained:

$$\begin{aligned} I(x,y) \; = \; & \frac{1}{Z}(4E(x,y) \; - \; E(x+\Delta,y) \; - \; E(x-\Delta,y) \\ & - \; E(x+\Delta,y+\Delta)E(x+\Delta,y-\Delta)) \\ & + \; \Delta YE(x,y) \end{aligned} \tag{1}$$

where ΔY is the shunt admittance.

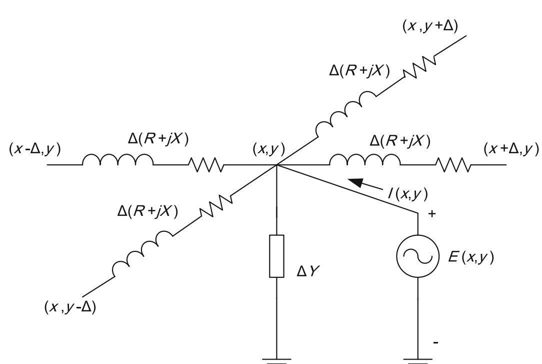

Fig. 1 Distributed power system model

Substituting the Taylor series expansions of $E(x+\Delta, y)$, $E(x-\Delta, y)$, $E(x, y+\Delta)$, and $E(x, y-\Delta)$ about $E(x, y)$ up to the fourth order into the above, (1) becomes

$$\begin{aligned} I(x,y) \; = \; & -\frac{\Delta^2}{Z}\left[\nabla^2 E(x,y) \; + \; \frac{\Delta^2}{12}\nabla^4 E(x,y)\right] \\ & + \; \Delta YE(x,y) \end{aligned} \tag{2}$$

Now, considering the source voltage as a constant in magnitude and a variable in phase δ, the power injecting into node (x, y) can be expressed as

$$P_e(x,y) \; = \; -\frac{\Delta^2 V^2}{|Z|^2}\left[x\nabla^2\delta \; - \; R(\nabla\delta)^2\right] \; + \; \Delta V^2 G \tag{3}$$

where $G = \mathrm{Re}\{Y\}$.

The classical model describing the electromechanical transients in power networks is the so-called swing equation. By introducing a set of units defined relative to a system base power S_{base}, the swing equation takes the following form:

$$\frac{2H}{\omega_0}\frac{d^2\delta}{dt^2} \; + \; \omega_0 D\frac{d\delta}{dt} \; = \; P_a \; = \; P_m \; - \; P_e \tag{4}$$

where H is the inertia constant in seconds; ω_0 is the rated electrical angular frequency or synchronous speed; D is the rotor damping constant in s^2; P_a is the accelerating power; P_m is the mechanical input power; P_e is the electrical output power; and δ is the rotor angle with respect to a synchronously rotating reference frame in electrical radians.

In the continuum model, the parameters of the swing equation are taken as the functions of the spatial coordinates x and y, which is:

$$H \;\rightarrow\; \Delta h(x,y) \quad D \;\rightarrow\; \Delta d(x,y)$$
$$Z \;\rightarrow\; \Delta z(x,y) \quad P_m \;\rightarrow\; \Delta p_m(x,y)$$

When Δ approaches to zero, the swing equation becomes:

$$\frac{\partial^2\delta}{\partial t^2} \; + \; \gamma\frac{\partial\delta}{\partial t} \; - \; v^2\nabla^2\delta \; + \; u^2(\nabla\delta)^2 \; = \; P \tag{5}$$

where $v^2 = \frac{\omega V^2 \sin\theta}{2h|z|}$; $u^2 = \frac{\omega V^2 \cos\theta}{2h|z|}$; $P = \frac{\omega\left(p_m - GV^2\right)}{2h}$; $\gamma = \frac{\omega^2 d}{2h}$; θ is the line impedance angle; V is the voltage magnitude; and G is the line conductance.

This is the so called continuum model. This model represents an alternative approach to analyse the electromechanical transients in power systems. The classical swing equation modelling the dynamics of rotors indirectly reflects the propagation of electromechanical transients in power networks suffering a disturbance. The continuum model assumes that each generator/load in a power system be represented as a continuum. The travelling solutions of this model demonstrate the disturbances of all generators, and directly show the propagation of electromechanical transients. The rationality of this partial differential equation model lies in the fact that the disturbances in realistic power networks propagate with a speed much less than light. Another benefit of this model involves the stability analysis. It is usually difficult to discuss the stability of generators one by one in multi-machine systems, since the number of generators could be very large. The continuum model overcomes this obstacle by the stability analysis of a continuum system.

2.1.2 The non-uniform continuum model

In [5], the per-unit length line impedances of transmission lines are assumed the same, and the orientation of the branches connecting the nodes allows for 0° or 90° with respect to the reference axis. Due to these assumptions the derived continuum model is of the so-called uniform form. In [6], these prerequisites are relaxed to reflect the anisotropy and heterogeneity. In turn, the countable finite branches connecting to the nodes allow for different per-unit length line impedances, and intersect with angle θ_i with respect to the reference axis.

Consider a line i connected to a generator. The two parts of this line which are at the left side and the right side of the x-axis are denoted as "−" and "+", respectively. The per-unit length line impedances of these two parts are $Z_i^- = R_i^- + jX_i^-$ and $Z_i^+ = R_i^+ + jX_i^+$, respectively. The angle θ_i of the line with respect to the reference axis is nonisotropic. $G_{int} - jB_{int}$ and $G_s - jB_s$ denote the per-unit length generator internal admittance and shunt admittance, respectively. The unit length of the line is Δ, and r_i is a unit vector. The internal and external voltage phase angles of the generator are $\phi(\bar{x})$ and $\delta(\bar{x})$, respectively. When Δ approaches to zero, the discrete system transforms into a so-called continuum system.

The power flow in line i can be obtained as

$$\begin{aligned}P_i^e = {} & \frac{\Delta R_i^+}{\Delta^2|Z_i^+|^2}\left[1-\cos(\delta(\bar{x})-\delta(\bar{x}+\Delta r_i))\right] \\ & + \frac{\Delta X_i^+}{\Delta^2|Z_i^+|^2}\sin(\delta(\bar{x}) - \delta(\bar{x}+\Delta r_i)) \\ & + \frac{\Delta R_i^-}{\Delta^2|Z_i^-|^2}\left[1 - \cos(\delta(\bar{x}) - \delta(\bar{x} - \Delta r_i))\right] \\ & + \frac{\Delta X_i^-}{\Delta^2|Z_i^-|^2}\sin(\delta(\bar{x}) - \delta(\bar{x}-\Delta r_i)) \end{aligned} \tag{6}$$

Given that the incremental unit Δ is small, by applying the Taylor series expansion, the following expressions can be obtained:

$$\begin{aligned}\delta(\bar{x}+\Delta r_i) = {} & \delta(\bar{x}) + [(r_i\cdot\overline{\nabla})\delta(\bar{x})]\Delta \\ & + \frac{1}{2}[(r_i\cdot\overline{\nabla})^2\delta(\bar{x})]\Delta^2 + o(\Delta^3)\end{aligned} \tag{7}$$

and

$$\begin{aligned}\delta(\bar{x}-\Delta r_i) = {} & \delta(\bar{x}) - [(r_i\cdot\overline{\nabla})\delta(\bar{x})]\Delta \\ & + \frac{1}{2}[(r_i\cdot\overline{\nabla})^2\delta(\bar{x})]\Delta^2 + o(\Delta^3)\end{aligned} \tag{8}$$

Using trigonometric equations for a sufficiently small Δ, (6) becomes

$$\begin{aligned}P_i^e = {} & -(r_i\cdot\overline{\nabla})[B_i(r_i\cdot\overline{\nabla})\delta]\Delta + G_i[(r_i\cdot\overline{\nabla})\delta]^2\Delta \\ & + o(\Delta^3)\end{aligned} \tag{9}$$

The above equation can be rewritten in a more compacted form as

$$P_i^e = [-\overline{\nabla}\cdot[B_i(\overline{\nabla}\delta)] + \overline{\nabla}\delta\cdot G_i\cdot\overline{\nabla}\delta]\Delta + o(\Delta^3) \tag{10}$$

where

$$G_i - jB_i = (G_i - jB_i)\begin{bmatrix}\cos^2\theta_i & \sin\theta_i\cos\theta_i \\ \sin\theta_i\cos\theta_i & \sin^2\theta_i\end{bmatrix} \tag{11}$$

Now assuming that there are N lines, then the total power injection into the network takes the following form:

$$\begin{aligned}P^e & = \sum_{i=1}^{N} P_i^e \\ & = [-\overline{\nabla}\cdot[B(\overline{\nabla}\delta)] + \overline{\nabla}\delta\cdot G\cdot\overline{\nabla}\delta]\Delta + o(\Delta^3)\end{aligned} \tag{12}$$

where

$$G - jB = \sum_{i=1}^{N}(\mathrm{G}_i - j\mathrm{B}_i)\begin{bmatrix}\cos^2\theta_i & \sin\theta_i\cos\theta_i \\ \sin\theta_i\cos\theta_i & \sin^2\theta_i\end{bmatrix} \tag{13}$$

The active power P^e supplied by the generator is

$$\begin{aligned}P^e = {} & \Delta G_{\mathrm{int}}[\cos(\delta(\bar{x}) - \phi(\bar{x})) - 1] \\ & - \Delta B_{\mathrm{int}}[\sin(\delta(\bar{x}) - \phi(\bar{x}))]\end{aligned} \tag{14}$$

Since the difference between the electrical and mechanical power causes the acceleration of the generator's rotor, the swing equation becomes

$$\Delta m\frac{\partial^2\phi}{\partial^2 t^2} + \Delta d\frac{\partial\phi}{\partial t} = \Delta p^m - P^e \tag{15}$$

The parameters m, d and p^m are the generator inertia, damping, and mechanical power, respectively.

Thus, the following equations are obtained:

$$\begin{aligned}-\overline{\nabla}\cdot\left[\mathrm{B}(\overline{\nabla}\delta)\right] + \overline{\nabla}\delta\cdot G\cdot\overline{\nabla}\delta = {} & G_{\mathrm{int}}[\cos(\delta-\phi)-1] \\ & - B_{\mathrm{int}}[\sin(\delta-\phi)] - G_S\end{aligned} \tag{16a}$$

$$\begin{aligned}m\frac{\partial^2\phi}{\partial t^2} + d\frac{\partial\phi}{\partial t} = {} & p^m - G_{\mathrm{int}}[1-\cos(\phi-\delta)] \\ & - B_{\mathrm{int}}[\sin(\phi-\delta)]\end{aligned} \tag{16b}$$

In practice, the structure of a power network is alike a mesh in the plane while generators and loads locate at discrete node in the network. An alternative is to consider the system parameters are smooth functions of spatial coordinates, which define a partial differential equation while they make the network structure preserved. Thus, the continuum model's parameters are parted into two groups: line parameters, such as G and B, describing a mesh in spatial coordinates; and nodal parameters, such as m, d, p^m, G_{int}, B_{int}, describing generator and loads in a power network [6].

Let the distributed real power be $p(\bar{x})$, then $P^e = \Delta p(\bar{x})$. So the following expression holds in spatial coordinates:

$$P(\bar{x}) = -\overline{\nabla} \cdot \left[B(\overline{\nabla}\delta)\right] + \overline{\nabla}\delta \cdot G \cdot \overline{\nabla}\delta \tag{17}$$

Since the per-unit length reactance is high with respect to resistance, namely $B \gg G$ in a lossless network or high voltage networks, the second term in the right hand of (17) can be neglected. Hence (17) reduces to:

$$P(\bar{x}) = -\overline{\nabla} \cdot \left[\mathrm{B}(\overline{\nabla}\delta)\right] \tag{18}$$

This equation embodies the significant insight of the power networks.

Let

$$p(\bar{x}) = -\left[\mathrm{B}(\overline{\nabla}\delta)\right] \tag{19}$$

then p is a vector field in the two dimensions plane. Compared with the conventional power flow equations, the vector field can be viewed as the active power flow density. Then, $p(\bar{x})$ measures the active power flow density that flows out of a point $\bar{x}$ in the network. Bearing the principle of continuity in physics in mind, which states that, in the absence of the creation or destruction of matter, the density within a region of space can change only by making it flow into or away from the region. It holds that if the divergence of p is positive at a point, the point is a source of power. Otherwise, if the divergence of the vector field is negative at the point or region, then the point or region is a sink.

Another interesting characteristic is that, if matrix B and G are assumed to be symmetrical second-order tensors, the anisotropic transmission lines can be analysed by tensor analysis. Reference [6] also demonstrates how these procedures proceed.

2.2 Wang's version

In [7], Delin Wang developed a similar continuum model in the simplest chain network with the same per-unit length admittance. The difference lies in that the continuum model is obtained through the wave equation of the forced torsional vibration equation of multi-mass disks in continuum. To that end, the electric power transmission through a transmission line is taken as the mechanical power through a shaft in multi-mass disks; a transmission line is treated as a shaft with elasticity but no inertia in a torsional vibration system. Bearing these similarities, the wave equation of electromechanical transients is derived. The correspondence between the variables in power systems and the variables in multi-mass disks is also discussed.

A classical forced torsional vibration system is composed of N disks with different inertia constants aligned in a straight light elastic shaft. For an ideal elastic shaft of length l without inertia and within its elastic limit, the following relationship of torque T and torsional angle θ holds

$$T = \frac{k}{l}\theta \tag{20}$$

where k is the torsional rigidity, and $k = \frac{1}{2}K\pi r^4$ when the elastic shaft is a solid cylinder with radius r and modulus with rigidity K.

Using Ω to represent the angular velocity of the shaft, then

$$P_m = T\Omega = \frac{k\Omega}{l}\theta \tag{21}$$

where P_m is the shaft power.

Labeling the internal damping coefficient as ε, the internal damping torque T_ε can be described as

$$T_\varepsilon = \frac{\varepsilon}{l}\theta^2 = \varepsilon l\left(\frac{\theta}{l}\right)^2 \tag{22}$$

Under some assumptions, the equation set describing the dynamics of the torsional vibration of multi-mass disks is

$$J_i'\frac{d^2\theta_i}{dt^2} + d_i'\frac{d\theta_i}{dt} = T_{i\Sigma}, \quad i = 1,2,\ldots,N \tag{23}$$

where θ_i is the torsional angle; J_i' is the moment of inertia of the i^{th} disk; d_i' is the damping coefficient; and $T_{i\Sigma}$ is the total driving torque imposed on the i^{th} disk.

If there are infinite disks spread along the finite length shaft, the distance between two disks can be considered as an infinitesimal. Therefore, the continuum model of torsional vibration of multi-mass disks is

$$J(x)\frac{\partial^2\theta}{\partial t^2} + d(x)\frac{\partial\theta}{\partial t} = k(x)\frac{\partial^2\theta}{\partial x^2} - \varepsilon(x)\left(\frac{\partial\theta}{\partial x}\right)^2 + T(x) \tag{24}$$

Consider the simplest chain system with each bus connected with a generator and a load, and buses connected by transmission lines in series. The length of a transmission line is l, the per-unit length impedance and the per-unit length admittance are $r_0 + jx_0$ and $g_0 - jb_0$, respectively. Then,

$$\begin{cases} G = g_0 l = \dfrac{g}{l} \\ B = b_0 l = \dfrac{b}{l} \end{cases} \tag{25}$$

The active power injected into the transmission line connecting the i^{th} and $(i+1)^{th}$ buses is

$$P = \frac{V_i V_{i+1}}{X} \sin \theta_{i,i+1} \tag{26}$$

where $\hat{V}_i = V_i \angle \theta_i$ and $\hat{V}_{i+1} = V_{i+1} \angle \theta_{i+1}$ are the bus voltages, $\theta_{i,i+1} = \theta_i - \theta_{i+1}$ is the voltage angle difference. Under certain assumptions, simplifications can be made:

$$P \approx \frac{1}{X}\theta_{i,i+1} \approx \frac{b}{l}\theta_{i,i+1} = B \cdot \theta_{i,i+1} \tag{27}$$

The line loss can be calculated as

$$\begin{aligned} P_r &= I^2 R = \frac{R}{R^2 + X^2}(\cos\theta_{i,i+1} - 1 + j\sin\theta_{i,i+1})^2 \\ &\approx \frac{g}{l}\theta_{i,i+1}^2 = gl\left(\frac{\theta_{i,i+1}}{l}\right)^2 \end{aligned} \tag{28}$$

Comparing (21) with (27), and (22) with (28), it is obvious that the electrical power flow in a transmission line has the same mathematical form as the mechanical power through a shaft. Thus, the chain power system is mathematically equivalent to the forced torsional vibration system of multi-mass disks, and the wave equation of the electromechanical disturbance propagation in the power system yields directly from (24) as

$$M(x)\frac{\partial^2\theta}{\partial t^2} + d(x)\frac{\partial\theta}{\partial t} = b(x)\frac{\partial^2\theta}{\partial x^2} - g(x)\left(\frac{\partial\theta}{\partial x}\right)^2 + p_m(x) \tag{29}$$

The per-unit form of (29) is

$$\frac{2h}{\omega}\frac{\partial^2\theta}{\partial t^2} + d\frac{\partial\theta}{\partial t} = b\frac{\partial^2\theta}{\partial x^2} - g\left(\frac{\partial\theta}{\partial x}\right)^2 + p_m \tag{30}$$

where h and d are the distributed inertia and the damping constant, respectively, and ω is the electrical angular velocity.

3 Analysis technique

3.1 Quantitative analysis

In [8], Delin Wang employed the WKBJ to approximate the analytical solutions of a non-uniform continuum model.

The dynamic behaviours describing the generator rotor's relative angular speed ω and the electrical power p are given by

$$\begin{cases} \dfrac{\partial^2\omega(x,t)}{\partial t^2} = \dfrac{1}{m(x)}\dfrac{\partial}{\partial x}\left[b(x)\dfrac{\partial\omega(x,t)}{\partial x}\right] \\ \dfrac{\partial^2 p(x,t)}{\partial t^2} = b(x)\dfrac{\partial}{\partial x}\left[\dfrac{1}{m(x)}\dfrac{\partial p(x,t)}{\partial x}\right] \end{cases} \tag{31}$$

where $m(x)$ is the per-unit rotor's inertia, and $b(x)$ is the per-unit length line susceptance. x represents an one-dimension spatial coordinate.

Let ω denote the rotor's relative angular speed, and let

$$\begin{cases} \omega(x,t) = \Omega(x)e^{j\varpi t} \\ p(x,t) = P(x)e^{j\varpi t} \end{cases} \tag{32}$$

where $\Omega(x)$ and $P(x)$ are complex functions. Then,

$$\begin{cases} \dfrac{1}{m(x)}\dfrac{d}{dx}\left[b(x)\dfrac{d\omega(x,t)}{dx}\right] + \varpi\Omega(x) = 0 \\ b(x)\dfrac{d}{dx}\left[\dfrac{1}{m(x)}\dfrac{dP(x,t)}{dx}\right] + \varpi P(x) = 0 \end{cases} \tag{32}$$

Assume that $b(x)$ and $m(x)$ are continuous functions of the spatial coordinate x, and the following expressions hold.

$$\begin{cases} \left|\dfrac{m'(x)}{m(x)}\right| = 1 \\ \left|\dfrac{m''(x)}{m'(x)}\right| = 1 \\ \left|\dfrac{b'(x)}{b(x)}\right| = 1 \\ \left|\dfrac{b''(x)}{b'(x)}\right| = 1 \end{cases} \tag{33}$$

Then, equation (32) becomes approximately

$$\begin{cases} \dfrac{d^2}{dx^2}\Omega(x) + \dfrac{\varpi^2}{v^2(x)}\Omega(x) \approx 0 \\ \dfrac{d^2}{dx^2}P(x) + \dfrac{\varpi^2}{v^2(x)}P(x) \approx 0 \end{cases} \tag{34}$$

where $v(x) = \sqrt{b(x)/m(x)}$ is the propagation speed of the wave crest.

Following the WKBJ approximate method of the classical wave motion theory in the non-uniform media, let

$$\begin{cases} \Omega(x) = \Omega_0(x) \cdot \exp[-jM(x)] \\ P(x) = P_0(x) \cdot \exp[-jN(x)] \end{cases} \tag{35}$$

Then, the WKBJ solution in the form of (32) can be obtained as

$$\begin{cases} \omega(x,t) = \Omega_0(x_0)\sqrt{\dfrac{v(x)}{v(x_0)}} \cdot \exp\left[j\varpi\left(t \mp \displaystyle\int_{x_0}^{x}\frac{1}{v(\xi)}d\xi\right)\right] \\ p(x,t) = P_0(x_0)\sqrt{\dfrac{v(x)}{v(x_0)}} \cdot \exp\left[j\varpi\left(t \mp \displaystyle\int_{x_0}^{x}\frac{1}{v(\xi)}d\xi\right)\right] \end{cases} \tag{36}$$

These are the solutions when $d^2\Omega_0(x)/dx^2$ and $d^2P_0(x)/dx^2$ are neglected.

It is difficult to integrate the analytical solution of the continuum model because its coefficients are functions of spatial coordinates. But if these coefficients are assumed to be constants, an analytical solution can be obtained, and to some extent explains the behaviour of power networks. However, this method ignores some characteristics of the power network. New tools, such as the Green function [9–11] or other numerical method, need to be employed to solve the partial differential equations. A simplified method to integrate the continuum equations numerically is presented in [12], where the concepts and tools in electromagnetic equations, such as the finite-differencing scenario, are applied to the solution of continuum power networks. By doing this, software designed for electromagnetics can be used for continuum power systems.

This method of using commercial software leads to several advantages, among them increased flexibility and reduced computational time.

3.2 Qualitative analysis

In [13], an analytical approach is proposed to study the propagation mechanism of electro-mechanical disturbances in a chain power system. Applying the Laplace transform to a mulit-segment uniform chain discrete power system can reveal significant characteristics of electromechanical disturbance propagation. Specifically, the analytical expression with respect to time, the reflection and transmission formulations can be carried out.

A mulit-segment uniform chain discrete power system includes two parts, parts I and II. Both parts are composed of the cascades of multiple T-type links. Part I consists of a transmission line with resistance $R_1/2$, reactance $X_1/2$, and a generator with rotor inertia M_1 and damping D_1. Part II consists of a transmission line with resistance $R_2/2$, reactance $X_2/2$, and a generator with rotor inertia M_2 and damping D_2.

Provided that there are q buses connected to a generator with rotor inertia constant M_1 and damping D_1, then the swing equations of Part I at buses k and $k+1$ can be expressed as

$$\begin{cases} M_1\dfrac{d}{dt}\varpi_k^I + D_1\varpi_k^I = -(p_k^I - p_{k-1}^I) \\ M_1\dfrac{d}{dt}\varpi_{k+1}^I + D_1\varpi_{k+1}^I = -(p_{k+1}^I - p_k^I) \end{cases} \tag{37}$$

where $1 \le k < k+1 < m$.

Using the Laplace transform yields

$$Z_2P_{k-1}^I - (Z_1 + 2Z_2)P_k^I + Z_2P_{k+1}^I = 0 \tag{38}$$

where $Z_1 = R_1^{'}+sX_1$, and $Z_2 = 1/(D_1 + sM_1)$.

Similarly, for Part II we have

$$Z_4P_{k-1}^{II} - (Z_3 + 2Z_4)P_k^{II} + Z_4P_{k+1}^{II} = 0 \tag{39}$$

where $Z_3 = R_2^{'} + sX_2$, $Z_4 = 1/(D_2 + sM_2)$ and $m+1 \le k < k+1 < q$.

Let the trial solution of incremental power on the transmission line between buses k and $k+1$ be

$$\begin{cases} P_k^I = A_1e^{-kv_1} + A_2e^{kv_1} \\ P_k^{II} = A_3e^{-kv_2} \end{cases} \tag{40}$$

where A_1, A_2, A_3, v_1 and v_2 are unknown complex constants.

Thus, the following results can be obtained:

$$\begin{cases} A_1 = \dfrac{E}{Z_2\sinh v_1(1+n_2e^{-2mv_1})} \\ A_2 = -n_2e^{-2mv_1}A_1 \\ A_3 = \dfrac{E\cdot\eta_2e^{-m(v_1-v_2)}}{Z_4\sinh v_2(1+n_2e^{-2mv_1})} \end{cases} \tag{41}$$

$$\begin{cases} n_2 = \dfrac{Z_4\sinh v_2 - Z_2\sinh v_1}{Z_4\sinh v_2 + Z_2\sinh v_1} \\ \eta_2 = \dfrac{2Z_4\sinh v_2}{Z_4\sinh v_2 + Z_2\sinh v_1} \end{cases} \tag{42}$$

Let n_2 and η_2 be the reflection coefficient and transmission coefficient of Part I, respectively. According to these equations, we have

$$\begin{cases} P_k^I = \dfrac{E}{Z_2\sinh v_1}\cdot\dfrac{e^{-kv_1} - n_2e^{-2mv_1}e^{kv_1}}{1+n_2e^{-2mv_1}} \\ \omega_k^I = [P_{k-1}^I - P_k^I]Z_2 = \dfrac{E\cdot(1-e^{-v_1})}{\mathrm{sh}v} \\ \quad\cdot\dfrac{e^{-(k-1)v_1} + n_2e^{-2mv_1}e^{kv_1}}{1+n_2e^{-2mv_1}} \end{cases} \tag{43}$$

where ω_k^I is the angular velocity at bus k in Part I.

Taking the similar procedures for Part II, we have

$$\begin{cases} P_k^{II} = \dfrac{E}{Z_4\mathrm{sh}v_2}\cdot\dfrac{\eta_2e^{-mv_1}e^{-(k-m)v_2}}{1-n_1n_2e^{-2mv_1}} \\ \omega_k^{II} = [P_{k-1}^{II} - P_k^{II}]Z_4 = \dfrac{E\cdot(e^{v_2}-1)}{\mathrm{sh}v_2}\cdot\dfrac{\eta_2e^{-(k-m)v_2}e^{-mv_1}}{1-n_1n_2e^{-2mv_1}} \end{cases} \tag{44}$$

where ω_k^{II} is the angular velocity at bus k in Part II.

Let the reflection coefficient be a real number. By neglecting the rotor damping coefficient D and the line equivalent loss coefficient and performing the inverse Laplace transform, the following expressions hold:

$$\begin{cases} P_{Bk}^I(t) = 2\dfrac{1}{X_1}\cdot E[J_{2k} - n_2J_{2(2m-k)} - n_2J_{2(2m+k)} \\ \qquad + n_2^2J_{2(4m-k)} + n_2^2J_{2(4m+k)} - n_2^3J_{2(6m-k)} - \cdots] \\ \tilde{\omega}_{Bk}^{\,I}(t) = 2E\sqrt{\dfrac{B_1}{M_1}}[J_{2k-1} + n_2J_{2(2m-k)+1} \\ \qquad - n_2J_{2(2m+k)+1} - n_2^2J_{2(4m-k)+1} \\ \qquad + n_2^2J_{2(4m+k)-1} + n_2^3J_{2(6m-k)+1} - \cdots] \end{cases} \tag{45}$$

$$\begin{cases} P_{Bk}^{II}(t) = 2\eta_2 \cdot \dfrac{E}{X_2}[J_{2k} - n_2 J_{2(2m+k)} + n_2^2 J_{2(4m+k)} + \cdots] \\ \tilde{\omega}_{Bk}^{II}(t) = 2\eta_2 E\sqrt{\dfrac{B_1}{M_1}}[J_{2k-1} - n_2 J_{2(2m+k)-1} \\ \qquad + n_2^2 J_{2(4m+k)-1} - \cdots] \end{cases} \tag{46}$$

Equations (45) and (46) reveal a significant characteristic that a linear combination of different orders of the Bessel function can represent the analytical solution to the equations governing electromechanical disturbance propagation in a multi-segment uniform system. Hence, the angle increment of the rotor could be rewritten as

$$\theta_k(t) = \sum_{i=0}^{\infty} C_{k,i} J_{f(i)}(\lambda_k t) \tag{47}$$

where f is a positive integer function, and $\{\lambda_k\}$ is a linear base of the Hilbert space.

Considering the properties of the Bessel function, the average velocities of the electromechanical disturbance propagation have the following relationships:

$$\begin{cases} v_I = \sqrt{\dfrac{B_1}{M_1}} \\ v_{II} = \sqrt{\dfrac{B_2}{M_2}} \end{cases} \tag{48}$$

An alternative method of analysing the continuum model is the so-called Lyapunov first method, also known as the small signal analysis method. However, a weakness of this method is that simplification of some nonlinear factors is inevitable.

4 Applications: zero reflection controller

Since electromechanical disturbances propagate as travelling waves, a controller oriented from impedance matching on transmission lines is constructed to quench the caused disturbances. In [14, 15], zero-refection controllers based on an impedance matching constraint at the boundary of the network dramatically suppress the propagation of the electromechanical waves.

4.1 The principle of zero reflection controllers

In [16], a one-dimension network with homogeneous lossless transmission lines is considered, and the damp of the generator's rotor is neglected. Under some certain assumptions, which are studied detailedly in [16], the following equations are obtained:

$$\frac{2h}{\omega_s}\frac{\partial\omega}{\partial t} = p_g - \frac{\partial P}{\partial x} \tag{49}$$

$$\frac{\partial P}{\partial t} = -E^2 b\frac{\partial\omega}{\partial x} \tag{50}$$

where $z^{-1} = g - jb$. Z is the impedance in per unit length; h is the inertia constant per unit length; d is the damping coefficient per unit length; the voltage magnitude E is constant in space and time.

Comparing (49) and (50) with the standard telegrapher's equations for electromagnetic waves on a transmission line, it is self-evident that P plays the role of line current and ω plays the role of voltage. Besides, linear undamped wave equations hold:

$$\frac{2h}{\omega_s}\frac{\partial^2\delta}{\partial t^2} = p_g + E^2 b\frac{\partial^2\delta}{\partial x^2} \tag{51}$$

With regard to a two-dimensional network, a similar derivation lead to the same type of equations, in which the Laplacian operator replaces the second derivative with respect to x. However, the units of the parameters are now different. Fortunately, a realistic power system could be thought of an interconnection of various one-dimensional lines.

Equation (51) is a wave equation, and the characteristic propagation velocity is given in the follow form:

$$v = \sqrt{(\omega V^2 \sin\theta)/2h|z|} \tag{52}$$

In [14], a string system with 64 identical generators connected to uniform transmission lines is studied to investigate the wave travelling in one-dimension power systems. The forward and reverse propagating waves and their reflections at the open-ended boundaries of the string of generators can be observed. The slow dispersion of the wave attributes to the generator damping and the dissipative component of the line impedance. It can be demonstrated that the theoretical value of the propagating time of the electromechanical waves agrees well with the numerically result [14].

The formulation of calculating wave length λ associated with the wave-like swing motions is given:

$$\lambda = \frac{v}{f} \tag{53}$$

where f is the frequency of the swing mode driving the wave-like propagation.

The electromechanical waves are analogous to travelling electromagnetic waves on transmission lines. As for transmission lines, current and voltage are used to describe the travelling waves, and the reflection is characterized in term of reflection coefficients in the light of the characteristic impedance of the lines. Additionally, proper termination can set the refection coefficient to zero, which excludes any refection on the line.

In the case of the power system model, which displays electromechanical wave phenomena, the variables frequency and power flow naturally play a similar part as voltage and current in the electromechanical transmission line case do. To associate the characteristic impedance with P and ω, Let the two variables regarding forward travelling waves be:

$$P^{+} = P(t - x/v) = P(y) \tag{54}$$

$$\omega^{+} = \omega(t - x/v) = \omega(y) \tag{55}$$

where v is the propagation velocity. Substituting these into (50) yields:

$$\frac{\partial P^{+}}{\partial y} = \frac{E^2}{v}\frac{\partial \omega^{+}}{\partial y} \tag{56}$$

Straightforwardly, we have:

$$P^{+} = \frac{E^2 b}{v}\omega^{+} \tag{57}$$

This implies that the forward propagating waves in term of power flow and frequency remain in a constant proportion. The characteristic impedance, or proportionality constant, is then defined by

$$C_o = \frac{\omega^{+}}{P^{+}} = \frac{v}{E^2 b} \tag{58}$$

At the termination of the string, the forward and reverse travelling waves must sum in a particular way to satisfy the imposed boundary condition. For example, if one termination is open, power flow is equal to zero, then the sum of the reverse travelling wave and the forward travelling wave must be zero at the boundary which means they are opposite number. This case is equivalent to a negative reflection in power flow and a positive reflection in frequency. If the string of generators is connected to a so-called infinite bus at which the frequency remains constant, then the forward travelling wave in frequency must be a negative reflection of the reverse travelling wave. This is equivalent to a positive reflection for power flow and a negative reflection in frequency.

4.2 The zero reflection control

Based on an approach used in the classical electromagnetic transmission line theory, if assume a time harmonic electromechanical wave excitation, then the time-harmonic telegrapher's equations is employed, which are ordinary differential equations in x for the $\omega(x)$ and $P(x)$. Thus, the solution of this equation can be derived for the frequency and power, respectively. In turn, these results can be used to calculate the reflection coefficient and transmission coefficient easily.

The typical case dealt with in the electromagnetic transmission line theory addressed only one transmission line ended with the load impedance C. Matching this impedance with the characteristic impedance yields zero reflection.

Changing the parameters of C, ω_C, and Pc results in various kinds of electromechanical wave controllers. These control laws stem from the aforementioned electromechanical wave theory but will be realized on discrete nonuniform power networks, where the wave behaviour might not exist apparently. The zero-transmission controller aims at quenching the perturbation in [16].

In [14], it is shown that how this impedance matching scheme can be implemented with the nonlinear discrete swing equation of a generator.

The similarity of electric power P and frequency ω to current and voltage in the transmission line theory, respectively, yields:

$$P = \frac{\omega}{C} \tag{59}$$

which is the controlled variable.

The control law

$$P_g = K\left(P - \frac{\omega}{C}\right) - P + D\omega \tag{60}$$

is proposed to design the zero-reflection control. Here, K is a gain, P_g is the deviation of P from its nominal value. This controller can be implemented if the parameters C and D need be estimated properly, and P and ω be measured accurately.

Simulation results show that this control law achieves to some extent the eradication of the reflection observed in the string system. In fact, the controller does not completely exclude reflections. One of the explanations is attributed to the difference between the wave equation model used to derive the controller and the simulation model. The former is continuum, linear, and neglects damping; the latter is discrete and of nonlinearities. Besides, the implementation of the controller is to feedback the control signal to an input into a generator, not achieved directly.

It is also possible to design a zero-reflection controller by manipulating the power flowing into the terminal load in a string of generators, and a method for this objective is proposed in [17].

4.3 Estimating the generation trip event

Since an electromechanical disturbance is propagating as a wave, a generation trip event can be estimated by synchronized phasor measurements [20].

Let the location of a frequency measurement unit and the detection time be (x_i, y_i) and t_i, respectively. The index i (i = 1,2,…,N) denotes the i^{th} frequency measurement unit deployed in the monitoring area. Let (x_e, y_e) and t_e represent the location and time of an event, respectively. Now the values of (x_e, y_e) and t_e need to be estimated so as to obtain the information corresponding to the event. Let the distance between the event and the i^{th} measurement unit take the following form:

$$L_i^2 = (x_i - x_e)^2 + (y_i - y_e)^2 \tag{61}$$

Denoting the average speed of the electromechanical wave as V, L_i can be expressed as

$$L_i = V(t_i - t_e) \tag{62}$$

Substituting (62) into (61), then there will be three unknown variables. If the number of the frequency measurement units is more than four, then the above two equations can be solved in light of the linear square method. The value of (x_e, y_e) and t_e can be obtained next.

5 Concluding remarks

In recent years, electromechanical wave propagation in power systems has received more and more concerns. In this paper, a brief description on the major versions of continuum models is first given. One of the principal goals is to demonstrate the applicability of the relevant concepts and approaches developed in other fields. For this purpose, some representative examples like adopting tensor analysis to explain the equation describing the structure of the network equation are investigated in detail, and the derivation of the continuum model from the multi-mass disks, the numerical solution of a continuum model by the method in electromagnetic equations, and the analytical solution of the continuum model in the WKBJ approximation. It is also discussed on how to employ traditional tools, such as small signal analysis to address the continuum model. Finally, some applications of the zero reflection controller based on continuum model are investigated to mitigate the electromechanical disturbances.

Unfortunately, the well-established small signal stability analysis method cannot accommodate the nonlinear term in the continuum model, which is of importance for describing the behaviour of power networks. Extending the research work towards the qualitative analysis of continuum models is highly expected.

One of the most striking applications of the continuum model in power networks is the electromechanical wave controllers aiming at reducing electromechanical disturbances, such as the zero reflection controllers. However, the influence of those controllers on power systems remains unclear. In [17], the stability of power networks with electromechanical wave controllers installed is discussed. In [19], the power system stabilizer is employed to mitigate the electromechanical wave. It is hoped that future efforts will reveal the relationship between the stability and these controllers from the perspective of control theory.

The issue that to what extent the continuum model accurately describes the behaviour of electromechanical transients in power systems is still an open question. In addition, it is possible that, through comparisons with other fields, the continuum model can be understood from various points of view. Recently, Nutaro and Protopoopescu [18] and his co-workers discovered a phenomenon that the delay of phase and frequency in power systems can be explained by the classical swing equation and Kirchhoff equation. It is expected that a further investigation on this issue could provide some insight on the continuum model.

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References

[1] Faulk D, Murphy RJ (1994) Comanche peak unit No. 2 100% load rejection test-underfrequency and system phasors measured across TU electric power system. In: Proceedings of the 47th annual conference for protective relay engineers, College Station, 21–23 Mar 1994

[2] Semlyen A (1974) Analysis of disturbance propagation in power systems based on a homogeneous dynamic model. IEEE Trans Power Appl Syst 93(2):676–684

[3] Cresap PL, Hauer JF (1981) Emergence of a new swing mode in the western power system. IEEE Trans Power Appl Syst 100(4):2037–2045

[4] Dersin P, Levis AH (1984) Aggregate feasibility sets for large power networks. In: Proceedings of the 9th triennial world congress of IFAC, vol 4, Budapest, 2–6 July 1984, pp 2163–2168

[5] Thorp JS, Seyler CE, Phadke AG (1998) Electromechanical wave propagation in large electric power systems. IEEE Trans Circ Syst I 45(6):614–622

[6] Parashar M, Thorp JS, Seyler CE (2004) Continuum modeling of electromechanical dynamics in large-scale power systems. IEEE Trans Circ Syst I 51(9):1848–1858

[7] Wang D, Wang X, Thorp JS (2006) Study on electromechanical wave continuum model for power systems in mechanics. In: Proceedings of the 2006 IEEE power engineering society general meeting, Montreal, 18–22 June 2006, 9 pp

[8] Wang D, Wang X (2008) Analytical study on electromechanical wave propagation in a non-uniform continuum power system. In: Proceedings of the 3rd international conference on deregulation and restructuring and power technologies, Nanjing, 6–9 Apr 2008, pp 958–963

[9] Logan JD (1988) Applied partial differential equations. Springer, New York
[10] Kevorkian J (2000) Partial differential equations. Springer, New York
[11] Haberman R (1988) Elementary applied partial differential equations: with Fourier series and boundary value problem, 3rd edn. Prentice-Hall, Englewood Ciffs
[12] Thomas AJ, Mahajan SM (2009) Electromechanical wave analysis through transient magnetic modeling. IEEE Trans Power Deliv 24(4):2336–2343
[13] Wang D, Wang X (2012) Electromechanical disturbance propagation and oscillation in power systems. In: Proceedings of the 2012 IEEE power and energy society general meeting, San Diego, 22–26 July 2012, 5 pp
[14] Lesieutre BC, Scholtz E, Verghese GC (2002) A zero-reflection controller for electromechanical disturbances in power networks. In: Proceedings of the 2002 power system computation conference, Seville, 24–28 June 2002, pp 913–919
[15] Lesieutre B, Scholtz E, Verghese G (2002) Impedance matching controllers to extinguish electromechanical waves in power networks. In: Proceedings of the 2002 international conference on control applications, vol 1, Glasgow, 18–20 Sept 2002, pp 25–30
[16] Scholtz E (2004) Observer-based monitors and distributed wave controller for electromechanical disturbances in power systems. Ph.D Thesis, MIT, Cambridge
[17] Samulesson O (1997) Power system damping-structural aspects of controlling active power. Ph.D. Thesis, Lund University, Lund
[18] Nutaro J, Protopoopescu V (2012) A new model of frequency delay in power systems. IEEE Trans Circ Syst II 59(10):1–5
[19] Zhang P, Thorp JS, Wang X et al (2012) The effect of electromechanical wave controller on inter-area modes. In: Proceedings of the 2012 IEEE power and energy society general meeting, San Diego, 22–26 July 2012, 8 pp
[20] Ali M, Buisson J, Phulpin Y (2010) Improved control strategy to mitigate electromechanical wave propagation using PSS. In: Proceedings of the 15th IEEE mediterranean electrotechnical conference, Valletta, 26–28 Apr 2010, pp 35–40
[21] Zheng G, Liu Y, Radman G (2012) Wide area frequency based generation trip event location estimation. In: Proceedings of the 2012 IEEE power engineering society general meeting, San Diego, 22–26 July 2012, 6 pp

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